Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name : Advanced Real Analysis

Subject Code : 5SC03ARA1 **Branch: M.Sc. (Mathematics)**

Semester : 3 **Date : 27/11/2018** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Define: σ - finite measure **(01)**

b. Suppose $(X, \mathcal{A}, \mu)$ is a measure space and E is the set of σ - finite measure. Let F is a measurable subset of E then F is a set of σ - finite measure. **(02)**

c. Let $(X, \mathcal{A})$ be a measurable space if $\lambda \,\&\, \mu$ are two measure on $(X, \mathcal{A})$ with $\lambda \perp \mu$ and $\lambda << \mu$ then $\lambda = 0$. **(02)**

d. Define: Mutually singular measure **(02)**

Q-2 **Attempt all questions** **(14)**

a. Prove that $(R, \mathcal{M}, m)$ is a complete measure. Where R be a set of real numbers, $\mathcal{M}$ be the collection of Lebesgue measurable sets and m be Lebesgue measure. **(05)**

b. State and prove Beppo-levi's theorem. **(05)**

c. If $E_1, E_2 \in \mathcal{A}$ then show that $\mu(E_1 \Delta E_2) = 0 \Rightarrow \mu(E_1) = \mu(E_2)$. Moreover if μ is complete and $E_1 \in \mathcal{A}$ with $\mu(E_1 \Delta E_2) = 0$ then $E_2 \in \mathcal{A}$ **(04)**

OR

Q-2 **Attempt all questions** **(14)**

a. State and prove Lusin's theorem. **(08)**

b. Let $(X, \mathcal{A}, \mu)$ be a measure space. Let s and t are two non-negative simple measurable function on X then prove that $\int_X (s+t)\, d\mu = \int_X s\, d\mu + \int_X t\, d\mu$ **(06)**

Q-3		**Attempt all questions**	**(14)**
	a.	State and prove Hahn-Decomposition theorem.	**(07)**
	b.	State and prove Lebesgue Dominated Convergence theorem.	**(05)**
	c.	Define: Signed measure	**(02)**

OR

Q-3		**Attempt all questions**	**(14)**
	a.	State and prove Jordan Decomposition theorem.	**(09)**
	b.	Let E be measurable set with $0<\nu(E)<\infty$ then E contains a positive set A with $\nu(A)>0$.	**(05)**

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a.	Define: Product measure space	**(01)**
	b.	State Fubini's theorem.	**(02)**
	c.	Prove that $\|\alpha f\|_\infty = \|\alpha\|\|f\|_\infty$.	**(02)**
	d.	Suppose $f \in L^p(\mu)$ and $f = g$ a.e. and μ be a complete measure then $g \in L^p(\mu)$.	**(02)**
Q-5		**Attempt all questions**	**(14)**
	a.	State and prove Radon-Nikodym theorem.	**(10)**
	b.	Show that Holder's inequality is an equality if $\alpha\|f\|^p = \beta\|g\|^q$ a.e. on X, for some $\alpha, \beta \in R-\{0\}$.	**(04)**

OR

Q-5		**Attempt all questions**	**(14)**
	a.	State and prove Lebesgue Decomposition theorem.	**(08)**
	b.	State and prove Holder's inequality.	**(06)**
Q-6		**Attempt all questions**	**(14)**
	a.	Prove that L^p spaces are complete spaces.	**(08)**
	b.	Let X be a normed space then X is complete iff every absolutely summable series is summable.	**(06)**

OR

Q-6		**Attempt all Questions**	**(14)**
	a.	State and prove Caratheodory theorem.	**(08)**
	b.	Prove the set of all simple measurable function f vanishing outside a set of finite measure is dense in $L^p(\mu)$, where $1 \le p < \infty$.	**(06)**